Direct Force Control Mode for Dynamic Mode AFM

State Estimator:

When the tip taps on a sample surface, the tip-sample interaction force presents itself as a disturbance to the cantilever. The dynamical state vector of the cantilever, consisting of its tip position and velocity, is augmented to include this disturbance as an additional state variable. This augmented state vector along with the dynamic model of the cantilever is employed to construct a closed-loop observer that estimates the tip-sample interaction force as well as the tip position and velocity with a desired rate of convergence. The closed-loop observer is expressed in the discrete time domain as : $\hat{\mathbf{X}}(k+1) = A_d\hat{\mathbf{X}}(k) + B_d u_s(k) + L_p\left\{y(k) - C_d\hat{\mathbf{X}}(k)\right\}$ where k denotes the current discrete time, $\hat{\mathbf{X}}(k)$ is the augmented state vector composed of tip position, tip velocity and disturbance, $u_s(k)$ is the sinusoidal input force, $y(k)$ is the measured tip position, A_d , B_d and C_d are augmented system matrices and L_p is a feedback gain matrix.

The estimated disturbance represents the tip-sample interaction force that may include the contact repulsive force $\hat{f}_{\text{int}}(k)$ and a long range force $\hat{\ell}(k)$. Based on the tip position, the contact repulsive force of each tapping cycle can be extracted.

Sample surface Estimator:

The sample surface in each tapping cycle is estimated from the measured tip position.

Model-based predictor:

A model-based predictor is designed to plan and control the next tapping through controlling the tip-to-sample distance.

- **Prediction of the tip position of the next tapping cycle**

The state vector (tip position and tip velocity) of the next tapping cycle when assuming no contact is predicted based on a linear dynamic model of the cantilever along with the estimated current state vector $\hat{\mathbf{x}}(k)$, which consists of tip position and tip velocity, and input force $u(k)$ and is expressed as

$$\hat{\mathbf{x}}(k+n) = A_d^n \hat{\mathbf{x}}(k) + \sum_{i=0}^{n-1} A_d^{n-1-i} B_d u(k+i),$$

where n denotes the number of discrete preview steps which is large enough to make (k+n)th discrete time belong to the next tapping cycle. The input force $u(k)$ is expressed as the summation of the sinusoidal force input $u_s(k)$ and the estimated long range force $\hat{\ell}(k)$. For the input forces $u(k+i)$ of future discrete time ($i \geq 1$), a zero-order predictor is used and expressed as $u(k+i) \approx u(k+i-m)$ where m denotes the number of discrete time steps of one tapping cycle.

- **Prediction of the tip-sample interaction force**

Since tapping occurs near the lowest tip position of the cycle, all the forces except the tip-sample interaction force during the period of contact are lumped into ma_e, where m is the lumped mass of the cantilever and a_e is the expected acceleration of the tip at the lowest position when assuming no contact. By employing a spring force model (sample stiffness : k_s) for the tip-sample interaction along with the lumped force ma_e, ***the impulse strength*** is approximated to be $2m(-\upsilon_{in}) - ma_e\pi\sqrt{m/k_s}$ for the case in which the sample stiffness is greater than that of the cantilever, where υ_{in} is the velocity, with which the cantilever is incident on the sample surface. In addition, ***the peak tip-sample interaction*** is approximated to be $\sqrt{mk_s\upsilon_{in}^2 + (ma_e)^2} - ma_e$.

The acceleration a_e can be directly predicted from the predicted state vector $\hat{\mathbf{x}}(k+n)$. For the sample stiffness k_s, a calibrated value can be used. The velocity υ_{in} of the next tapping depends on the sample position $\hat{x}_s$, the z motion δz and the long range force besides the transient dynamic response of the cantilever. The predicted state vector $\hat{\mathbf{x}}(k+n)$ of the next tapping cycle includes the effect of latter two and thus, the velocity υ_{in} is approximated with the acceleration a_e and the expected lowest tip position $\hat{x}_m$ of the next tapping cycle (when assuming no contact) which are also directly calculated from $\hat{\mathbf{x}}(k+n)$ along with the sample position and the z motion and is expressed as $-\sqrt{2a_e\left\{\hat{x}_s-\left(\hat{x}_m+\delta z\right)\right\}}$. For the sample surface $\hat{x}_s$ of the next tapping cycle, a zero order predictor can be used.

Therefore, the impulse strength $\hat{p}$ of the next tapping can be expressed as

$$\hat{p}=P(\delta z)=2m\sqrt{2a_e\left\{\hat{x}_s-\left(\hat{x}_m+\delta z\right)\right\}}-ma_e\pi\sqrt{m/k_s}\ .$$

On the other hand, the peak tip-sample interaction force f_{peak} of the next tapping is expressed as

$$f_{peak}=F(\delta x_z)=\sqrt{2mk_sa_e\left\{\hat{x}_s-\left(\hat{x}_m+\delta z\right)\right\}+\left(ma_e\right)^2}-ma_e\ .$$

- **Desired z motion to regulate force**

  Finally, for a given reference impulse force p_r, the desired z motion δz which regulates the impulse force is calculated:

$$\delta z=P^{-1}(p_r)=\hat{x}_s-\hat{x}_m-\frac{1}{2a_e}\left\{\frac{1}{2m}\left(p_r+ma_e\pi\sqrt{m/k_s}\right)\right\}^2$$

For a given reference peak tip-sample interaction force f_{peak}^r, the desired z motion δz which regulates the peak force is calculated:

$$\delta z=F^{-1}(f_{peak}^r)=\hat{x}_s-\hat{x}_m-\frac{1}{2mk_sa_e}\left\{\left(f_{peak}^r\right)^2+2ma_e\cdot f_{peak}^r\right\}$$

<u>Feedback regulator:</u>

The variations in the sample material property and the prediction errors can cause errors in determining the desired z motion. A feedback regulator which uses the estimated tip-sample interaction force as the feedback signal is employed to attenuate these effects and directly control the tip-sample interaction force.

Summary

State Estimator

$$\hat{\mathbf{X}}(k+1) = A_d\hat{\mathbf{X}}(k) + B_d u_s(k) + L_p\left\{y(k) - C_d\hat{\mathbf{X}}(k)\right\}$$

State predictor of the next tapping cycle

$$\hat{\mathbf{x}}(k+n) = A_d^n\hat{\mathbf{x}}(k) + \sum_{i=0}^{n-1} A_d^{n-1-i} B_d u(k+i)$$

Desired z motion which regulates the impulse force

$$\delta z = P^{-1}(p_r) = \hat{x}_s - \hat{x}_m - \frac{1}{2a_e}\left\{\frac{1}{2m}\left(p_r + ma_e\pi\sqrt{m/k_s}\right)\right\}^2$$

Desired z motion which regulates the peak tip-sample interaction force

$$\delta z = F^{-1}(f_{peak}^r) = \hat{x}_s - \hat{x}_m - \frac{1}{2mk_s a_e}\left\{\left(f_{peak}^r\right)^2 + 2ma_e \cdot f_{peak}^r\right\}$$

Parameters calculated from state estimator

- $\hat{\mathbf{x}}(k)$: Current state vector (tip position and tip velocity)
- $\hat{f}_{\text{int}}(k)$: Contact repulsive force
- $\hat{\ell}(k)$: Long range force

Parameters calculated from state predictor

- $\hat{x}_m$: The expected lowest position of the tip of the next tapping cycle when assuming no contact
- a_e : The expected acceleration of the tip at the lowest position of the next tapping cycle when assuming no contact

Parameters predicted with a zero order predictor

- $u(k+i)$: force input when $i \geq 1$
- $\hat{x}_s$: the sample surface of the next tapping cycle

Parameter given with a calibrated value

- k_s : the sample stiffness